

Allometric Equations

Allometric equations describe the relationship of one part of a plant to another part of a plant. Usually some parts of the plant are easier to measure than other parts. We usually make measures that are easy to measure and predict the hard parts (eg. Use diameter to predict tree height for a given species).

In this unit we will learn about some common allometric equations and how to use them in our biometrics work.

A common task with allometric equations include, making a plot of the equation so that you can evaluate its performance against data or another equation. This exercise will step you through the process of evaluating equations and plotting them in R.

I have a website, (oak.snr.missouri.edu/sylvan/en/functions.php) that has a collection allometric equations from all over the world. The purpose is to provide a readily accessible resource for growth modeling projects. At this site we have equations for the following relationships:

- Maximum Crown width
- Height-Diameter
- Bark thickness
- Height-age
- Site index
- Maximum Stand Density Index

If you click on one of these you will see the following. I click on height-diameter View link.

Firefox | Height-Diameter Equations | Mizzou Advantage | University of Mis... | MIZZOU Magazine - Make a model, e... | Account Summary

oak.snr.missouri.edu/~emcda/loadPaper.php?file=htdbh.xml

Height-Diameter Equations

Dump all paper data into CSV file

AUTHORS | SPECIES

- Colbert, K. C., D. R. Larsen, and J. R. Lootens. 2002
- Donnelly, D., B. Lilly, and E. Smith. 2001
- Ek, A., 1974
- Hanus, M. L., D. D. Marshall, and D. W. Hann. 1999
- Huang, Shongming. 1999
- Larsen, D. R. and D. W. Hann. 1987
- Lootens, J. R., D. R. Larsen, S. R. Shifley. 2007
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Colbert, K. C., D. R. Larsen, and J. R. Lootens. 2002. Height-diameter equations for thirteen Midwestern bottomland hardwood species. *Northern Journal of Applied Forestry* 19 (4):171-176.
<http://www.treesearch.fs.fed.us/pubs/11791>

Region of Application: Iowa, Illinois, Missouri Bottomland

Function: $ht = bh + e^{p1+p2 dbh^{p3}}$

y Variable: ht (Height)

x1 Variable: bh (Breastheight)

x2 Variable: dbh (Diameter at breast height)

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 Return to top

Height (feet)

Diameter at Breast Height (inches)

graph	all	none	species	code	units	bh	p1	p2	p3	xmin	xmax	ymin	ymax	n
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	American elm	ULAM	english	4.5	5.79	-4.1352	-0.3485	0.3	25.2	6.5	98.4	222
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	Ash	FRAXI	english	4.5	5.23	-3.7257	-0.5013	0.3	25.4	6.5	144.4	110
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	Black willow	SANI	english	4.5	4.55	-3.7529	-0.9168	0.4	24.3	6.5	105.0	66
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	Boxelder	ACNE2	english	4.5	5.13	-3.5461	-0.4298	0.1	22.0	6.5	75.4	146
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	Eastern cottonwood	PODE3	english	4.5	6.45	-5.5	-0.7402	1.9	41.1	7.0	147.6	224
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	Hackberry	CEOC	english	4.5	6.29	-4.4757	-0.2702	0.1	19.5	6.5	85.3	310
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	Pin oak	QUPA2	english	4.5	5.68	-3.9049	-0.3965	0.6	30.3	6.5	114.8	122
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	Red mulberry	MORU2	english	4.5	45.3	-43.6193	-0.0199	0.2	12.4	6.5	72.2	171
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	Silver maple	ACSA2	english	4.5	5.07	-3.1207	-0.5272	0.3	38.7	6.5	141.1	823

3:53 PM
2/28/2012

Figure 1. The height-diameter equation papers at (oak.snr.missouri.edu/sylvan/functions.html) with the authors tab showing.

This is the information for the height diameter information in the data set. If you click on the authors tab, each author in the list is shown. The Colbert paper is the first one in the list and is shown in the bottom half of the screen. You can also click on the species tab and see.

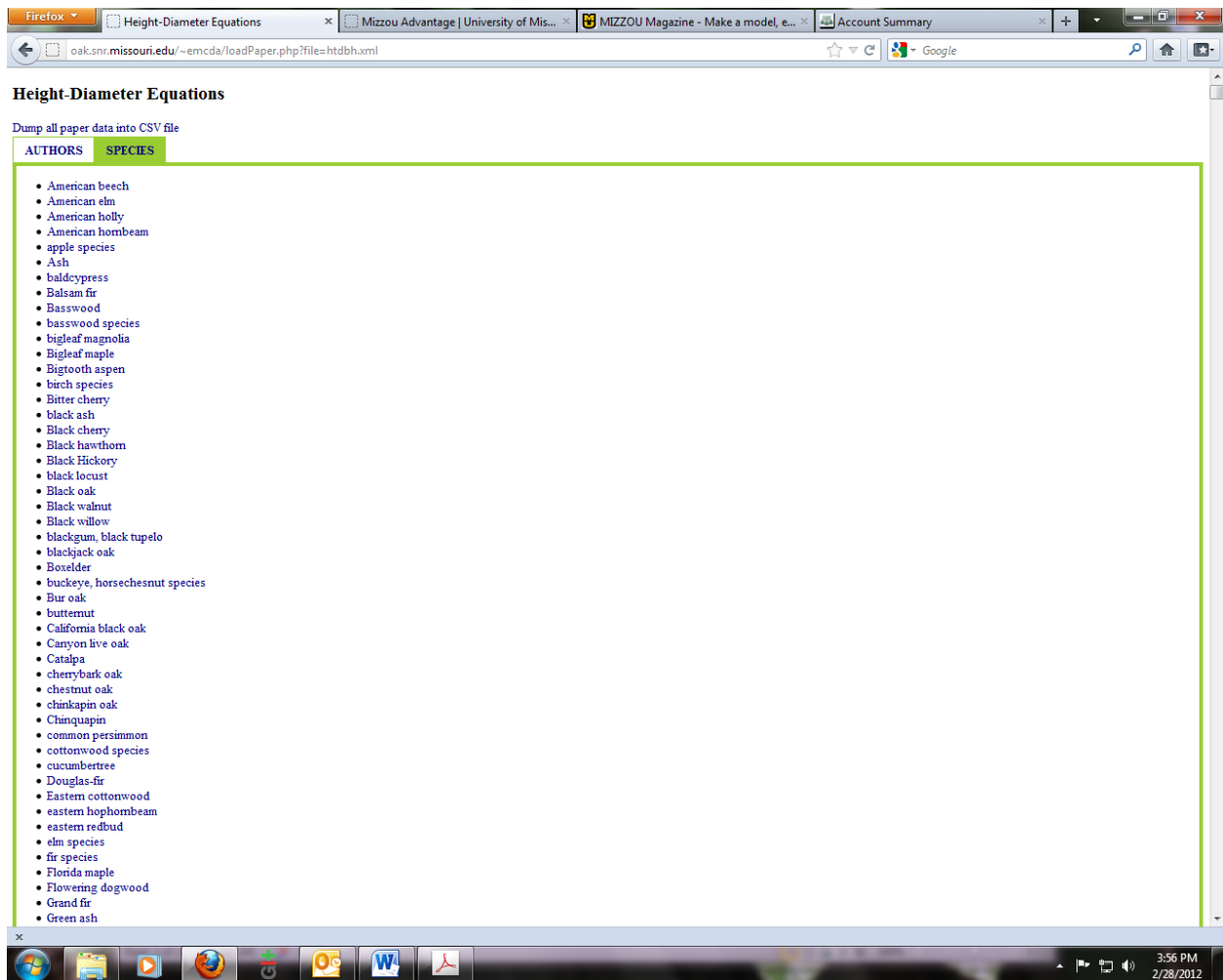


Figure 2. The height-diameter equation papers at (oak.snr.missouri.edu/sylvan/functions.html) with the species tab showing.

By scrolling down you will see all species available in the included papers for this particular relationship (height – diameter).

First the paper is cited as you would expect in a normal reference citation. Also there is a link to the location of a PDF of the paper. There is information on the relevant location for the equations. There is the equation and a list of the variable and their meanings. Also the files data can be dumped to a cvs file.

This particular webpage is interactive. This means that the equations can be drawn by your preferences. If you press the all at the top of the table you will see:

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<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	American elm	ULAM	english	4.5	5.79	-4.1352	-0.3485	0.3	25.2	6.5	98.4	222
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Region of Application: Southeastern US

4:07 PM 2/28/2012

Which plots all equations in the paper. Individual species can be turned on and off by check the box to the left of each line. The color of the box is the color of the line in the graph. This provides a quick what to compare functions. Now all the information you need to plot this line in R can be found on this page.

You must translate the equation in to an R equation.

$$ht = bh + e^{p1+p2dbh^3}$$

Becomes

```
>bh+exp(p1+p2*dbh^p3)
```

We can assign bh, p1, p2, and p3 or just put the number is the line as:

```
> 4.5+exp(5.07-3.1207*seq(1,38,0.1)^-0.5272)
```

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4:28 PM 2/28/2012

Figure 3. Here we have plotted height diameter equations for silver maple and sycamore within the range of the original data.

By using the steps in the video we can produce the following graph.

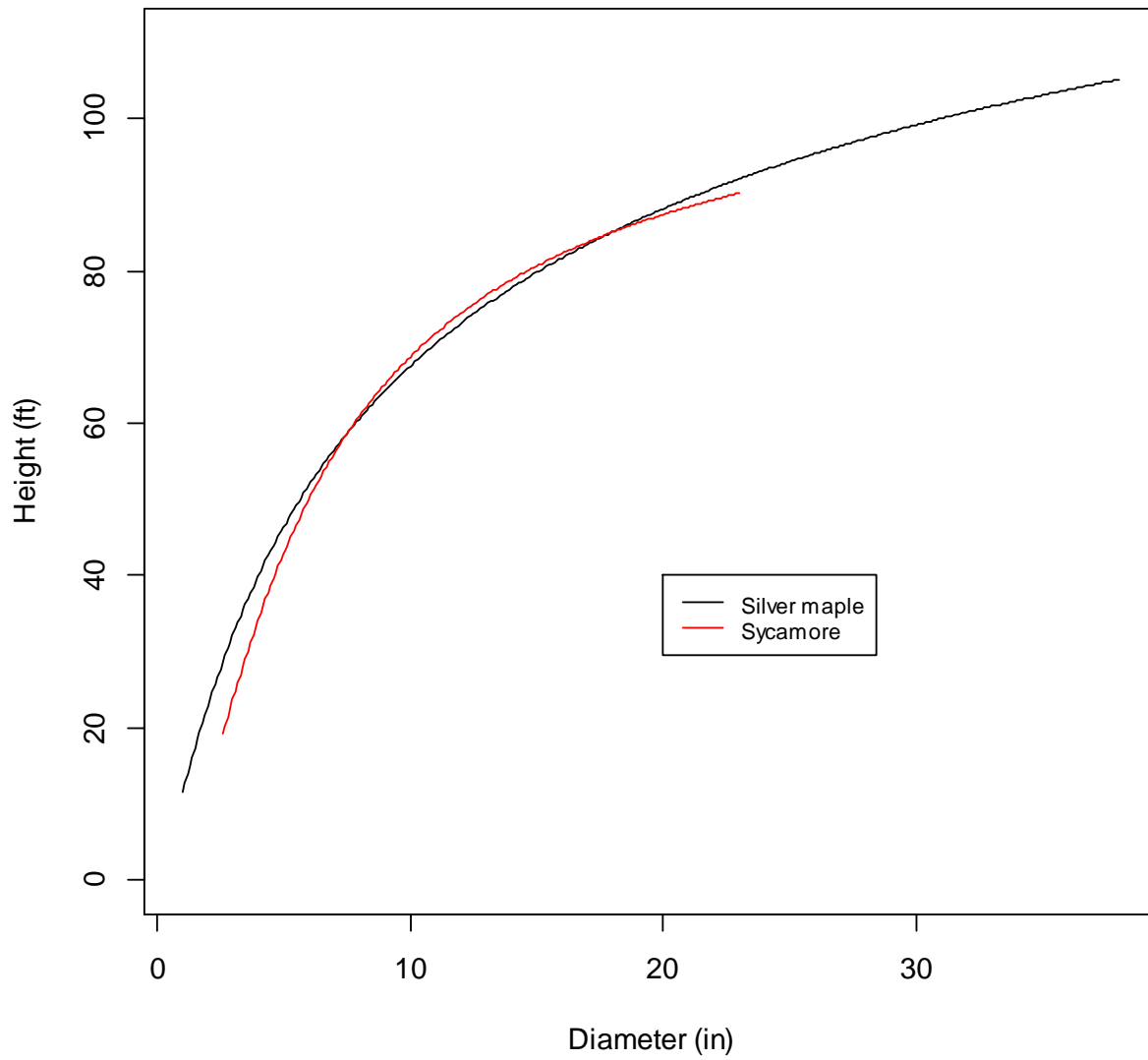


Figure 4. The R graph produced to compare the height diameter equations from silver maple and sycamore trees.