Schnabel's Mark and Recapture

The Schnabel method is an extension of the Petersen method to a series of samples. Each sampling event includes capture, examination for previous marks, marking and then release. The marking time is not need so only one type of mark is needed. For each sample we need to record the following data:

c_t= Total number of individuals caught in sample t
R_t = Number of individuals already marked when caught in sample
u_t= Number of individuals marked for the first time and released in sample t ,
Accidental deaths are subtracted.

Marks accumulate over time. So

M_t = the number marked individual in the population just before the current sample.

or as a formula

$$M_t = \sum_{i=1}^{t-1} U_i$$

where

S is the number of the sample in the series.

Schnabel (1938) proposed the following weighted average of Petersen estimates as an estimate of population density.

$$\hat{N} = \frac{\sum_{t=1}^{S} (C_t M_t)}{\sum_{t=1}^{S} R_t}$$

If the each sample catch and the marked population are less than 10 % of the population size the following is a better estimator of the population size.

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$$\hat{N} = \frac{\sum_{t=1}^{S}(C_t M_t)}{(\sum_{t=1}^{S} R_t) + 1}$$

where:

S is the number of samples in the series.

The variance estimator is calculated on the reciprocal of the population density 1/N as:

$$Variance\left(\frac{1}{\hat{N}}\right) = \frac{\sum_{t=1}^{S} R_t}{\sum_{t=1}^{S}(C_t M_t)^2}$$

And the standard error of the reciprocal population density is:

$$Standard\ error\left(\frac{1}{\hat{N}}\right) = \sqrt{Variance\left(\frac{1}{\hat{N}}\right)}$$

Schumacher and Eschmeyer (1943) pointed out that one could use a regression a slope of 1/N passing through the origin. A formula using the regression methodology is:

$$\hat{N} = \frac{\sum_{t=1}^{s}(C_t M_t^2)}{\sum_{t=1}^{s} R_t M_t}$$

The variance of this density estimate can be estimated from the variance estimator or the slope parameter estimate (Zar, 1996) as:

$$Variance\left(\frac{1}{\hat{N}}\right) = \frac{\sum_{t=1}^{S}\left(\frac{R_t^2}{C_t}\right) - \frac{\left(\sum_{t=1}^{S}(R_t M_t)\right)^2}{\sum_{t=1}^{S}(C_t M_t^2)}}{s-2}$$

And the standard error of the population density estimate can be determined by:

$$Standard\ error\left(\frac{1}{\hat{N}}\right) = \sqrt{\frac{Variance\left(\frac{1}{\hat{N}}\right)}{\sum_{t=1}^{s}(C_t M_t^2)}}$$

Confidence Intervals: If the total number of recapture in < 50 the confidence limits for the population estimate should be obtained from a Poisson distribution (Krebs 1989, Appendix 1.2). If the number of recaptures is >50 use the normal approximation as follows:

$$\frac{1}{\hat{N}} \pm t_\alpha s_{\bar{x}}$$

where:

S.E. is the standard error of 1/N
t alpha is the value from Student's t-table for (100 - alpha) % confidence limits.

Assumptions:

- Assumptions of the Petersen method apply.
- In the Schnabel method it is easier to pick up violation of the assumptions.

Also See:

Chapter 2 - Estimating Abundance: Mark-and-Recapture pages 30-37 in:

Krebs, C. J. 1998. Ecological Methodology. Harper and Row, Publishers. New York. 654 pp.

Schnabel, Z. E. 1938. The estimation of total fish populations of a lake. Am. Math. Monthly 45:348-352.

Schumacher, F. X. and Eschmeyer, R. W. 1943. The estimation of fish populations in lakes and ponds. J. Tennessee Acad. Sci. 18:228-249.

Zar, J. H. 2007. Biostatistical Analysis. Prentice-Hall, Inc. Englewood Cliffs, New Jersey. 718 pp.

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